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Oscillating modes of thermomagnetic avalanches in superconductors

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Abstract: In this work, the nature of the magnetic flux oscillations in superconductors was studied within the adiabatic approximation. It was shown that, under certain conditions, depending on the values of parameters of the system, oscillating modes of the magnetic flux may be observed.

Keywords: critical state; flow jumps; small perturbations; superconductors; vortex oscillations.

1 Introduction

The flux jumps results in a large-scale flux avalanches in a superconductor and their origin are related to the magnetothermal instabilities [1]. Conventionally, thermomagnetic instabilities were interpreted in terms of thermal runaway, triggered by local energy dissipation in the sample [2]. According to this theory, any local instability causes a small temperature rise, the critical current is decreased and magnetic flux moves much easily under the Lorentz force. The additional flux movement dissipates more energy further increasing temperature. This positive feedback loop may lead to a flux jumps in the superconductor sample. In [3] the phenomenon of magnetic flux oscillations has been detected, which arises as a result of thermomagnetic instability in superconductors. The observed oscillations were explained by the existence of a finite value of the effective vortex mass, i.e., oscillations can be considered as a manifestation of the inertial properties of vortex matter [4, 5]. In the present work, the dynamics of the magnetic flux oscillations in superconductors was studied within the adiabatic approximation in superconductors.

2 Basic equations

The system of equations of macroscopic electrodynamics is used to simulate the evolution of temperature and electromagnetic field perturbations. The distribution of magnetic induction $\vec{B}$ and transport current $\vec{j}$ in a superconductor is given by the equation

$$\nabla \times \vec{B} = \mu_0 \vec{j}. \tag{1}$$

The relationship between the magnetic induction $\vec{B}$ and electric field $\vec{E}$ is described by Maxwell's equations

$$\nabla \times \vec{E} = -\frac{d\vec{B}}{dt}, \tag{2}$$

$$\vec{E} = \vec{B} \times \vec{v}. \tag{3}$$

The equation of motion of the vortices can be written in the form

$$m\frac{d\vec{v}}{dt} + \eta\vec{v} + F_L + F_P = 0, \tag{4}$$

where m is the mass of the vortex of unit length, $\vec{F}_L = \vec{j} \times \Phi_0$ is the Lorentz force, $F_P = \Phi_0 \left[\vec{j} \times n\right]$ is the pinning force, $\eta = \frac{1}{2}\frac{\Phi_0}{\rho_n} B_{C2}$ is the viscosity coefficient, ρ_n is the resistance in the normal state, $\Phi_0 = \frac{h}{2e}$ is the magnetic flux quantum, $\vec{B}_{c2}$ is the upper critical field. In combining the relation (4) with Maxwell's Eqs. (1) and (2), we obtain a nonlinear diffusion equation for the magnetic flux induction $\vec{B}$ in the following form

$$\frac{d\vec{B}}{dt} = \nabla\left(\vec{v} \cdot \vec{B}\right), \tag{5}$$

$$m\frac{d\vec{v}}{dt} + \eta\vec{v} = -\Phi_0(\vec{j} - \vec{j}_c), \tag{6}$$

and the temperature distribution in superconductor is governed by the heat conduction diffusion equation

$$\nu\frac{dT}{d}t = \nabla\left[\kappa(T)\nabla T\right] + \vec{j}\vec{E}, \tag{7}$$

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where $v = v(T)$ and $\kappa = \kappa(T)$ are the heat capacity and thermal conductivity coefficients of the sample, respectively. We use the Bean model for the current density, which assumes that it does not depend on the magnetic field induction, $j = j_C(B_e, T)$, i.e., $j_C = j_0 - a(T - T_0)$, where B_e is the value of the external magnetic induction; $a = j_0/(T_C - T_0)$; j_0 is the equilibrium current density, T_0 and T_C are the initial and critical temperature of the sample, respectively [8]. Let us consider the superconducting semi-infinitive sample ($x \geq 0$). The magnetic field with the flux density $B(x, t)$ is applied in the direction of the z-axis, the transport current $j(x, t)$ and the electric field $E(x, t)$ are induced inside the slab along the y-axis [6].

3 Results and discussions

Let us present a solution of Eqs. (5)–(7) in the form

$$B = B_e(x) + b(x, t),$$
$$v = v_0(x) + v(x, t), \qquad (8)$$
$$T = T_0(x) + \Theta(x, t),$$

where $T_0(x)$, $B_e(x)$, and $v_0(x)$ are the solutions to the unperturbed equations, which can be obtained within a quasi-stationary approximation. Substituting the above solution (8) into Eqs. (5)–(7), we obtain the following system of differential equations

$$\frac{d\Theta}{d\tau} = 2v - \beta\Theta, \qquad (9)$$

$$\mu\frac{dv}{d\tau} + v = -\frac{db}{dz} + \beta\Theta, \qquad (10)$$

$$\frac{db}{d\tau} = \left(\frac{db}{dz} + b\right) + \left(\frac{dv}{dz} + v\right), \qquad (11)$$

where dimensionless parameters $\mu = \frac{\Phi_0}{\mu_0 \eta^2}\frac{B_e}{2L^2}$, $\beta = \frac{a}{v j_c}\frac{B_e^2}{\mu_0}$ and variables $b = \frac{B}{B_e} = \frac{1}{\mu_0}\frac{B_e}{j_C L}$, $\Theta = \frac{2\mu_0 v}{B_e^2}$, $\upsilon = v\frac{t_0}{L}$, $z = \frac{x}{L}$, $\tau = \frac{t}{t_0} = \frac{\Phi_0}{\mu_0^{2\eta}}\frac{B_e}{2j_C L^2}$ were introduced. Here, $L = \frac{1}{\mu_0}\frac{B_e}{j_C}$ is the depth of penetration of the magnetic field into the superconductor [6]. We present the solution of system (9)–(11) in the form of small magnetic and thermal perturbations $\Theta(z, \tau), b(z, \tau), v(z, \tau) \sim \exp(\gamma \cdot \tau)$, (where γ is the eigenvalue of the problem to be determined). Substituting the last solution to the system of (9)–(11) obtain the following dispersion relations to determine the eigenvalue problem Q5

$$\frac{d^2 b}{dz^2} - [(\gamma + \beta)\mu - 2\beta]\frac{db}{dz} + [(\mu + 1)\gamma^2 - 2\beta(\mu - 1)]\, b = 0. \qquad (12)$$

The instability of the magnetic front, as a rule [7], is determined by the positive values of the increment Re $\gamma \geq 0$. Analysis of the dispersion relation (12) shows that the increment is positive Re $\gamma \geq 0$, if the condition $\mu \geq \mu_C = 2$ is met. In this case $\mu \geq \mu_C$, the small perturbations increase with time and the magnetic flux front is unstable. In the opposite case ($\mu \leq \mu_C$) any small perturbation will decay, at $\mu = \mu_C$ the increment is zero ($\gamma = 0$). In particular case, when $\mu = 1$ the increment γ is determined by the stability parameter β, so the stability criterion can be presented as $\beta > 1$. In another particular case, when the thermal effects (adiabatic approximation [6]) are negligible ($\beta \ll 1$) we obtain the dispersion relation

$$\frac{d^2 b}{dx^2} - \mu\frac{db}{dx} + [(\gamma - 1)(\mu + 1)]\, b = 0 \qquad (13)$$

In this approximation, we present the solution of the dispersion Eq. (13) as $b \sim \exp(-i\mathbf{k}x)$, and obtain the dependence of the increment γ on the wave vector $\mathbf{k}$. An analysis shows, that when $\mathbf{k} < \mathbf{k}_c = \mu$, the increment is positive and the small perturbation increases with time. For the values of the wave vector $\mathbf{k} > \mathbf{k}_c$, the quantity γ is negative and the small perturbation decays exponentially. It can be shown, that for $\mathbf{k} = \mathbf{k}_c$ the increment is $\gamma = 0$. If the wave vector tends to zero $\mathbf{k} \to 0$ or infinity $\mathbf{k} \to \infty$, the quantity $\gamma = 1$ and a small perturbation increases.

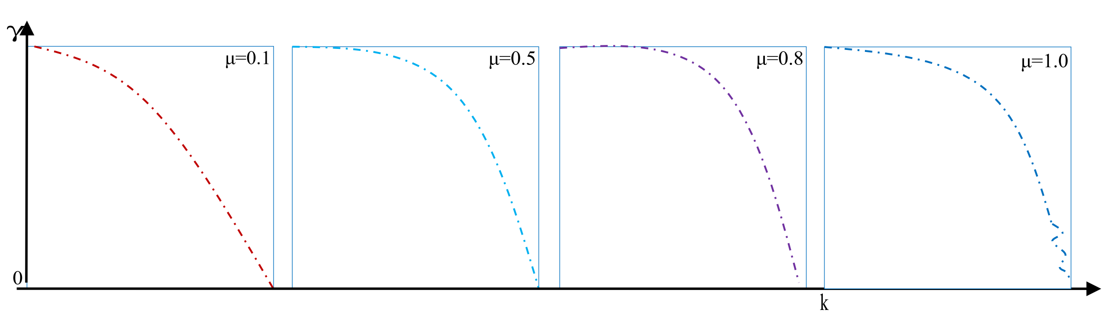

Figure 1: The dependence of the growth rate of γ on the wave vector $\mathbf{k}$ for $\mu = 0.1, 0.5, 0$ and 1.0.

For $\mu = 0$, the wave vector $\mathbf{k} = 0$, for the value of $\gamma = 1$, and it increases if $\gamma > 1$, and decreases in opposite case $\gamma < 1$. The dependence of the growth rate of γ on the wave vector is shown in Figure 1 for various values of the parameter μ. As μ increases, the parameter $\mathbf{k}$ increases. The magnetic flux oscillations is observed at values of the parameter $\mu \sim 1$, which take into account the inertial properties of the vortices.

4 Conclusions

In conclusion, the spatial and temporal distributions of small thermal and electromagnetic perturbations in a plane semi-infinite superconducting sample are studied. Based on the system of equations for the temperature, magnetic induction, and vortex motion, a dispersion relation was obtained. It was shown that, under certain conditions, depending on the values of the parameters of the system oscillation modes of the magnetic flux may be observed.

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Q6
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