

# PHYSICAL SCIENCES

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## Contents

### General

Jie Liao, Wenlin Feng and Xiaozhan Yang  
**Simultaneous demodulation comparison of fiber-optic Fabry–Perot sensors connected in parallel and series — 875**

Lian Wang, Juncheng Zhou, Yuhao Chen, Liu Xiao, Guojia Huang, Xinyue Huang and Xiaozhan Yang  
**An intensity modulated fiber-optic carbon monoxide sensor based on Ag/Co-MOF *in-situ* coated thin-core fiber — 881**

### Dynamical Systems & Nonlinear Phenomena

Muhammad Khalid, Mohsin Khan, Muddusir, Ata-ur-Rahman and Muhammad Irshad  
**Periodic and localized structures in dusty plasma with Kaniadakis distribution — 891**

Jyoti Wadhwa and Arvinder Singh  
**Study of optical guiding of the Hermite–Gaussian laser beam in preformed collisional parabolic plasma channel and second harmonic generation — 899**

Amit K. Pal  
**Stability analysis of a delayed predator–prey model with nonlinear harvesting efforts using imprecise biological parameters — 909**

### Hydrodynamics

Chunling Fan, Jiangfan Qin, Qihua Fan and Chuntang Zhang  
**Gas–liquid two-phase flow pattern analysis based on multiscale symbolic transfer entropy — 923**

Ahcene Nouar, Amar Dib, Mohamed Kezzar, Mohamed R. Sari and Mohamed R. Eid  
**Numerical treatment of squeezing unsteady nanofluid flow using optimized stochastic algorithm — 933**

### Solid State Physics & Materials Science

Pokkunuri Pardhasaradhi, Boddapati Taraka Phani Madhav, Gandu Srilekha, Manepalli Ramakrishna Nanchara Rao and Gorla Venkata Ganesh  
**Identification of thermo optical parameters in 4<sup>1</sup>-hexyloxy-4-cyanobiphenyl with dispersed ZnO nano particles — 947**

Taylanov Nizom Abdurazzakovich, Bekmirzaeva Xursand, Urozov Abduxolik Nurmamatovich and Igamqulova Zilola  
**The process of magnetic flux penetration into superconductors — 959**

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- [Contents](#)
- [Journal Overview](#)

[Accessible](#) October 5, 2021

### Frontmatter

Page range: i-iii

Cite this [Download PDF](#)

General

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### **Simultaneous demodulation comparison of fiber-optic Fabry–Perot sensors connected in parallel and series**

Jie Liao, Wenlin Feng, Xiaozhan Yang

Page range: 875-880

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Abstract

In this work, the spectra of two fiber-optic Fabry–Perot sensors in parallel and series connection were studied. The spectrum of the parallel structure is a simple superposition of the two sensors' spectrum, and that of the series structure can be regarded as the interference occurring in two Fabry–Perot sensors successively. The sensors' optical path difference can be obtained and separated by using the theoretical formula to fit the normalized spectrum of parallel or series structure, which showed that two or more Fabry–Perot sensors can be simultaneously demodulated by the spectrum fitting method.

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### **An intensity modulated fiber-optic carbon monoxide sensor based on Ag/Co-MOF *in-situ* coated thin-core fiber**

Lian Wang, Juncheng Zhou, Yuhao Chen, Liu Xiao, Guojia Huang, Xinyue Huang, Xiaozhan Yang

Page range: 881-889



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### Abstract

An intensity modulated fiber-optic carbon monoxide (CO) sensor by integrating in-situ solvothermal-growth Ag/Co-MOF sensing film is fabricated and evaluated. The Michelson interference sensing structure is composed of single-mode fiber (SMF), enlarged taper, thin-core fiber (TCF), and Ag film as the reflector. Ag/Co-MOF was coated on the cladding of the TCF as the sensing material, and the enlarged taper is located between TCF and SMF as the coupler. The structure, morphology, compositions and thermal stability of the Ag/Co-MOF sensing film were characterized by X-ray diffraction (XRD), scanning electron microscopy (SEM), transmission electron microscopy (TEM), X-ray photoelectron spectroscopy (XPS), thermogravimetric analysis (TGA), etc. The sensitivity of the sensor is 0.04515 dB/ppm, and the fitting parameter of the CO concentration is 0.99876. In addition, the sensor has the advantages of good selectivity, good signal and temperature stability, and it has potential application in trace CO detection.

Dynamical Systems & Nonlinear Phenomena

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## Periodic and localized structures in dusty plasma with Kaniadakis distribution

Muhammad Khalid, Mohsin Khan, Muddusir, Ata-ur-Rahman, Muhammad Irshad

Page range: 891-897

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### Abstract

The propagation of electrostatic dust-ion-acoustic nonlinear periodic waves is investigated in dusty plasma wherein electrons follow Kaniadakis distribution. The Korteweg–de Vries (KdV) and modified Korteweg–de Vries (mKdV) equations are derived by employing reductive perturbation method and their cnoidal wave solutions are analysed. The effect of relevant parameters (viz.,  $\kappa$ -deformed parameter  $\kappa$  and dust concentration  $\beta$ ) on the dynamics of cnoidal structures is discussed. Further it is found that amplitude of compressive cnoidal waves increases with increasing values of  $\beta$ , while reverse effect is observed in case of rarefactive cnoidal structures with rising values of  $\beta$ . Also  $\kappa$ -deformed parameter  $\kappa$  bears no effect on cnoidal waves associated with KdV equation, whereas  $\kappa$ -deformed parameter  $\kappa$  significantly affects the cnoidal waves associated with mKdV equation.

Requires Authentication [Accessible](#) July 29, 2021

## Study of optical guiding of the Hermite–Gaussian laser beam in preformed collisional parabolic plasma channel and second harmonic generation

Jyoti Wadhwa, Arvinder Singh

Page range: 899-908

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### Abstract

In the present work, the scheme of optical guiding of the Hermite–Gaussian laser beam and the generation of second-harmonic  $2\omega$  radiation ( $\omega$  being the frequency of incident beam) is presented in plasma having the preformed collisional plasma channel in which density variation is parabolic. The nonlinear coupling of excited electron plasma wave with the carrier or incident

beam results in the production of second harmonics of the latter. The method of moments is used for finding the coupled differential equations for the beam diameter to study the dynamics of the Hermite–Gaussian laser beam in plasma under the effect of the collisional parabolic channel. For numerical simulations, the Runge–Kutta fourth-order numerical method is used. Standard perturbation theory gives the equation for excitation of electron plasma wave which further acts as the source term for the second harmonic generation. The numerical results show that the preformed plasma channel has a significant effect on the guiding as well as on the  $2\omega$  generation of the Hermite–Gaussian laser beam in plasma.

Requires Authentication [Accessible](#) July 13, 2021

### **Stability analysis of a delayed predator–prey model with nonlinear harvesting efforts using imprecise biological parameters**

Amit K. Pal

Page range: 909-921

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Abstract

In this paper, the dynamical behaviors of a delayed predator–prey model (PPM) with nonlinear harvesting efforts by using imprecise biological parameters are studied. A method is proposed to handle these imprecise parameters by using a parametric form of interval numbers. The proposed PPM is presented with Crowley–Martin type of predation and Michaelis–Menten type prey harvesting. The existence of various equilibrium points and the stability of the system at these equilibrium points are investigated. Analytical study reveals that the delay model exhibits a stable limit cycle oscillation. Computer simulations are carried out to illustrate the main analytical findings.

Hydrodynamics

Requires Authentication [Accessible](#) August 23, 2021

### **Gas–liquid two-phase flow pattern analysis based on multiscale symbolic transfer entropy**

Chunling Fan, Jiangfan Qin, Qihua Fan, Chuntang Zhang

Page range: 923-932

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Abstract

This paper presents a multiscale symbolic transfer entropy (MSTE) to extract the features of gas–liquid two-phase flow and distinguish flow patterns effectively. The role of the MSTE in typical chaotic time series is investigated. Then the characteristics of the flow patterns about three gas–liquid two-phase flows are analyzed from the perspective of causal analysis. The results show that the MSTE can identify different flow patterns and characterize the dynamic characteristics of flow patterns, providing a new method for identifying two-phase flow accurately. In addition, the MSTE reduces the influence of noise to a certain extent and preserves the dynamic characteristics based on simplifying the original sequence. Compared with traditional algorithm, the MSTE has fast calculation speed and anti-interference characteristics and can express the essential features well.

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## **Numerical treatment of squeezing unsteady nanofluid flow using optimized stochastic algorithm**

Ahcene Nouar, Amar Dib, Mohamed Kezzar, Mohamed R. Sari, Mohamed R. Eid

Page range: 933-946

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[Abstract](#)

In this paper, very efficient, intelligent techniques have been used to solve the fourth-order nonlinear ordinary differential equations arising from squeezing unsteady nanofluid flow. The activation functions used to develop the three models are log-sigmoid, radial basis, and tan-sigmoid. The neural network of each scheme is optimized with the interior point method (IPM) to find the weights of the networks. The confrontation of the obtained results with the numerical solutions shows good accuracy of the three schemes. The obtained solutions by utilizing the neural network technique of our variables field (velocity and temperature) are continuous contrary to the discrete form obtained by the numerical scheme.

Solid State Physics & Materials Science

Requires Authentication [Accessible](#) July 21, 2021

## **Identification of thermo optical parameters in 4'-hexyloxy-4-cyanobiphenyl with dispersed ZnO nano particles**

Pokkunuri Pardhasaradhi, Boddapati Taraka Phani Madhav, Gandu Srilekha, Manepalli

Ramakrishna Nanchara Rao, Gorla Venkata Ganesh

Page range: 947-957

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[Abstract](#)

In this present article, synthesis, characterization, and study of optical parameters through image enhancement methods have been carried out on 4'-hexyloxy-4-cyanobiphenyl (6OCB) pure liquid crystal (LC) and 6OCB with dispersion of 0.5 wt% ZnO nanoparticles. Textural determinations of the synthesized compounds are recorded by using SDTECHS POM connected with a hot stage and camera. differential scanning calorimetry (DSC) is used to measure enthalpy and transition temperature values. The results show that the dispersion of ZnO in 6OCB exhibits nematic phase as same as the pure 6OCB with slightly reduced clearing temperature as expected. Further characterization is carried out by various spectroscopic techniques like X-ray diffraction (XRD), scanning electron microscopy (SEM), and ultraviolet visible (UV-Vis) spectroscopy. To evaluate and identify the behavior of optical parameters viz optical transmittance (OT), absorption coefficient (AC), and phase retardation (PR) as a function of temperature, an image processing method has been proposed, i.e. illumination enhancement algorithm (IEA) using MATLAB software. The proposed enhancement algorithm is one of the simplest and efficient algorithms to evaluate the thermo optical parameters for various electro optical applications. The results are compared with the body of the data available.

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## **The process of magnetic flux penetration into superconductors**

Taylanov Nizom Abdurazzakovich, Bekmirzaeva Xursand, Urozov Abduxolikh Nurmatovich,  
Igamqulova Zilola

Page range: 959-963

[More](#)

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Taylanov Nizom Abdurazzakovich\*, Bekmirzaeva Xursand, Urozov Abduxolik Nurmamatovich and Igamqulova Zilola

# The process of magnetic flux penetration into superconductors

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**Abstract:** In the present paper the magnetic flux penetration dynamics of type-II superconductors in the flux creep regime is studied by analytically solving the nonlinear diffusion equation for the magnetic flux induction, assuming that an applied field parallel to the surface of the sample and using a power-law dependence of the differential resistivity on the magnetic field induction. An exact solution of nonlinear diffusion equation for the magnetic induction  $B(r, t)$  is obtained by using a well-known self-similar technique. We study the problem in the framework of a macroscopic approach, in which all length scales are larger than the flux-line spacing; thus, the superconductor is considered as a uniform medium.

**Keywords:** critical state; magnetic flux penetration; nonlinear diffusion.

## 1 Introduction

Theoretical investigations of the magnetic flux penetration dynamics into superconductors in various regimes with various current–voltage characteristics is one of the key problems of electrodynamics of superconductors. The mathematical problem of theoretical study of the dynamics of evolution and penetration of magnetic flux into the sample in the viscous flux flow regime can be formulated on the basis of a nonlinear diffusion-like equation [1] for the magnetic field induction in a superconductor. The dynamics of space-time evolution of the magnetic flux penetration into type-II superconductors, where the flux lines are parallel to the surface of the sample for the viscous flux flow regime with a nonlinear relationship between the field and

current density in type II superconductors, has been studied by many authors [2–4]. The magnetic flux penetration problem for the particular case, where the flux flow resistivity is independent of the magnetic field was theoretically studied in [2]. A similar problem has been considered in [3] for the semi-infinite sample in parallel geometry.

The magnetic flux penetration into the superconductor sample, where the flux lines are perpendicular to the surface of the sample is described by a non-local nonlinear diffusion equation. This problem has been exactly solved by Briksin and Dorogovstev [4] for the case thin film geometry in the flux flow regime of type-II superconductors. In the present paper the dynamics of the magnetic flux penetration in the flux creep regime of type-II superconductors is studied. The nonlinear diffusion equation for the magnetic flux induction is solved. It is assumed that a field is applied parallel to the surface of the sample. An exact solution of nonlinear diffusion equation for the magnetic induction  $B(\vec{r}, t)$  is obtained by using a well-known self-similar technique [2]. We study the problem in the framework of a macroscopic approach, in which all length scales are larger than the flux-line spacing; thus, the superconductor is considered as an uniform medium.

## 2 Formulation of the problem

Bean [5] has proposed the critical state model which is successfully used to describe magnetic properties of type II superconductors. According to this model, the distribution of the magnetic flux density  $B$  and the transport current density  $\vec{j}$  inside a superconductor is given by a solution of the equation

$$\text{rot } \vec{B} = \frac{4\pi}{c} \vec{j}. \quad (1)$$

When the penetrated magnetic flux changes with time, an electric field  $\vec{E}(r, t)$  is generated inside the sample according to Faraday's law

$$\text{rot } \vec{E} = -\frac{1}{c} \frac{d\vec{B}}{dt}. \quad (2)$$

The electric field  $\vec{E}(r, t)$  induced by the moving vortices is related to the magnetic flux induction  $\vec{B}(r, t)$  by the

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following relation

$$\vec{E} = \frac{U}{c} \vec{B}. \quad (3)$$

In general, the dynamics of magnetic flux penetration process is determined by many external and internal factors, as the sweep rate of the external magnetic field, the type of voltage–current characteristics, the critical current density and its magnetic field and temperature derivatives, the profile temperature and surface cooling conditions, the sample geometry and pinning properties of the considered sample. Many results on this problem, have been obtained in the flux flow regime, where voltage current–current characteristics of superconductor is described by linear dependence of  $\vec{j}(\vec{E})$  at sufficiently large values of electric field [5]. The nonlinear part of the curve  $\vec{j}(\vec{E})$  in the region of weak electric fields is associated with flux creep. In the flux creep regime the magnitude of flux penetration profile strongly depends on variation of external parameters, in particular, on the magnetic field sweep rate. The current–voltage characteristics of type-II conventional as well as high- $T_c$  superconductors in the flux creep regime is a highly nonlinear due to thermally activated magnetic flux motion. Thermally activated flux motion or flux creep problem in superconductor samples with various geometries and conditions has been recently extensively studied in (see [3]). According to Kim–Anderson model [6] the velocity of the thermally activated flux motion can be presented as

$$v = v_0 \exp\left(-\frac{U}{k_B T}\right). \quad (4)$$

Here  $v_0$  is the velocity of the thermally activated flux motion at zero temperature  $T = 0$ ,  $U$  is the activation energy due to vortex pinning, which depends on temperature  $T$ , magnetic field induction  $B$  and current density  $j$ ;  $U(\vec{j}) = U(\vec{j}, \vec{B}, T)$ . For the simple case it can be presented as [6]

$$U(\vec{j}) = U_0 \left(1 - \frac{\vec{j}}{j_c}\right), \quad (5)$$

where  $U_0$  is the characteristic scale of the activation energy and  $\vec{j} = j_c(\vec{B})$  is the critical current density. In the flux creep state the effective activation energy  $U$  grows logarithmically [2] with decreasing current density as

$$U(\vec{j}) = U_c \ln\left(\frac{j_c}{\vec{j}}\right)^n, \quad (6)$$

where the exponent  $n$  depends upon the flux creep regime. The expression (6) gives a quite realistic description for activation barriers in a wide range of temperatures and magnetic fields. Taking into account Eqs. (3) and (4) the

phenomenological relation  $\vec{E}(\vec{j})$  may be chosen in the power-law form

$$\vec{E} = v_0 \vec{B} \cdot \left(\frac{\vec{j}}{j_c}\right)^n, \quad (7)$$

where  $n = \frac{U_0}{k_B T}$  is a function of temperature  $T$ , magnetic field  $H$  and depends on the pinning regimes of superconductors. In such case an analytical solution of the nonlinear creep equation can be constructed by choosing the critical current dependence on the magnetic field. Many models have been for the functional form of  $\vec{j}_c(\vec{B})$ . For the critical current we adopt the power-law model [3], which can be applied over a relatively wide magnetic field range except in the high field region near the upper critical field

$$\vec{j}_c(\vec{B}) = j_{01} \left(\frac{B_0}{B}\right)^\gamma, \quad (8)$$

where  $j_{01}$  and  $B_0$  are the characteristic values of the current density and magnetic field induction, respectively;  $\gamma$  is the dimensionless pinning parameter, usually  $0 < \gamma < 1$ . If we assume  $\gamma = 0$ , the above model reduces to the Bean–London model [5]. This model is applicable to the case where  $\vec{j}_c$  can be regarded approximately field independent. Another possible decay law would be exponential, which has often been used to take into account its decrease with the magnetic field

$$\vec{j}_c(\vec{B}) = j_1 \exp\left(-\frac{B}{B_1}\right), \quad (9)$$

where  $j_1$  and  $B_1$  are the phenomenological parameters related to the pinning ability: the smaller  $B_1$ , the more drastic, is the decrease of the critical current with field. The numerical methods have been applied to resolve the flux diffusion equation, employing the exponential critical state model. Next, based on the power law and exponential models, we shall study the distribution of the magnetic induction, current density and magnetization of superconductors.

### 3 Basic equations

We formulate the general equation governing the dynamics of the magnetic field induction in a superconductor sample. We study the evolution of the magnetic penetration process in simple geometry – a superconducting semi-infinite sample  $x \geq 0$ . We assume that the external magnetic field induction  $\vec{B} = (0, 0, B_z)$  is parallel to the  $z$ -axis. When the magnetic field with the flux density  $\vec{B}(r, t)$

is applied in the direction of the  $z$ -axis, the transport current  $\vec{j}(r, t)$  and the electric field  $\vec{E} = (0, E_e, 0)$  are induced inside the slab along the  $y$ -axis. For this geometry, the spatial and temporal evolution of magnetic field induction  $\vec{B}(r, t)$  is described by the following nonlinear diffusion equation in the generalized dimensionless form [2]

$$\frac{db}{dt} = \frac{d}{dt} \left[ b^{\gamma+1} \left| \frac{db}{dr} \right|^{n-1} \frac{db}{dr} \right], \quad (10)$$

where we have introduced the dimensionless variables  $b = B/B_0$ ,  $t = t/\tau_0$ ,  $j = j/j_0$ ,  $e = E/v_0 j_0$ , and parameters  $x_p = \frac{\mu_0 j_0}{B_0} x$ ,  $B_0 = \mu_0 j_0 v_0 \tau$ . The diffusion Eq. (10) can be integrated analytically subject to appropriate initial and boundary conditions in the center of the sample and on the sample's edges. We consider the case when the magnetic field applied to sample increases with time according to a power law with the exponent of  $\alpha > 0$

$$b(0, t) = b_0 t^\alpha. \quad (11)$$

where  $b_0$  is the constant parameter. The boundary condition (11) is equivalent to a linear increase of the magnetic field with time, which corresponds to a real experimental situation. As can be easily seen that the case  $\alpha = 0$  describes a constant applied magnetic field at the surface of the sample, while the case  $\alpha = 1$  corresponds to linearly increasing applied field, respectively [3]. The other boundary condition follows from the continuity of the flux at the free boundary  $x = x_p$

$$b(x_p, 0) = 0, \quad (12)$$

where  $x_p$  is the dimensionless position of the front of the magnetic field. The flux conservation condition for the magnetic field induction can be formulated in the following integral form

$$\int b(x, 0) dx = 1. \quad (13)$$

It should be noted that the nonlinear diffusion Eq. (10), completed by the boundary conditions (11)–(13) for magnetic induction, totally determines the problem of the space-time distribution of the magnetic flux penetration into superconductor sample in the flux creep regime with a power-law dependence of differential resistivity on the magnetic field induction. Solution of this equation gives a complete description of the time and space evolution of the magnetic flux in a sample.

## 4 Scaling solution

Now, we show that the nonlinear diffusion Eq. (10) can be solved exactly, using well-known scaling methods [1, 2]. We present the solution of nonlinear diffusion equation for the magnetic induction (10) in the following scaling form

$$b(x, t) = t^\alpha f(x/t^\beta). \quad (14)$$

The similarity exponents  $\alpha$  and  $\beta$  are of primary physical importance since the parameter  $\alpha$  represents the rate of decay of the magnetic induction  $b(x, t)$ , while the parameter  $\beta$  is the rate of spread of the space distribution as time goes on. Inserting this scaling form into differential Eq. (10) and comparing powers of  $t$  in all terms, we get the following relationship for the exponents  $\alpha$  and  $\beta$ . Using the condition of the flux conservation (13) we obtain

$$\alpha = \beta = 1/(2n + \gamma n + 1), \quad (15)$$

which suggests the existence of self-similar solutions in the form

$$b(x, t) = t^{1/(2n+\gamma n+1)} f(z), \quad z = xt^{1/(2n+\gamma n+1)}. \quad (16)$$

Substituting this scaling solution (16) into the governing Eq. (10) yields an ordinary differential equation for the function  $f(z)$  in the form

$$\frac{d}{dz} \left[ f^{\gamma+1} \left| \frac{df}{dz} \right|^n \right] + \frac{1}{2n + \gamma n + 1} \frac{d}{dz} \left[ z \frac{df}{dz} \right] = 0. \quad (17)$$

The boundary conditions for the function  $f(z)$  now become

$$f(0, t) = 1, \quad f(z_0, t) = 0. \quad (18)$$

Eq. (17), depending on the initial and the boundary conditions, describes a scaling-like behavior magnetic flux front with a time-dependent velocity in the sample. After further integration and applying the boundary conditions (18) we get the following solution of the problem

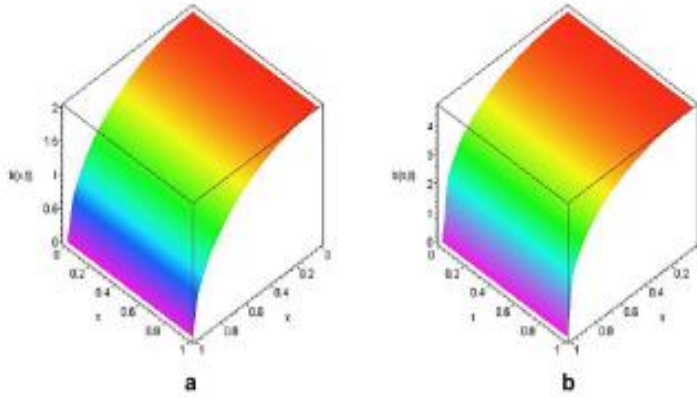
$$f(z) = f(z_0) \left[ 1 - (z/z_0)^{(n+1)/n} \right]^{1/(\gamma+1)}, \quad (19)$$

where

$$f(z_0) = \left[ \frac{n\gamma+1}{n+1} \left( \frac{z_0^{n+1}}{2n+\gamma n+1} \right)^{1/n} \right]^{1/(\gamma+1)}.$$

The position of the front  $z_0$  can now be found by substituting the solution (19) into the integral condition (13) and it is given by





**Figure 1:** (a) and (b) The distributions of the normalized flux density  $b(x, t)$  at time  $t = 1$ , for  $\gamma = 1, 2$ .

$$z_0^{(2n+\gamma n+1)/(\gamma+1)} = \left[ \frac{\frac{n}{n+1} F\left(\frac{\gamma+2}{\gamma+1}, \frac{1}{2}\right)}{\Gamma\left(\frac{\gamma+2}{\gamma+1}\right) \Gamma\left(\frac{n}{n+1}\right)} \right] \times \left[ n \frac{\gamma+1}{n+1} \left( \frac{1}{2n+\gamma n+1} \right)^{1/n} \right]^{1/(\gamma+1)}.$$

It is convenient to write the self-similar solution (18) in terms of primitive variables as

$$b(x, t) = b_0 \left[ 1 - \left( \frac{x}{x_p} \right)^{(n+1)/n} \right]^{1/(\gamma+1)}, \quad (20)$$

where

$$\begin{aligned} b_0(0, t) &= b(x, t) \\ &= t^{-1/(2n+\gamma n+1)} \\ &\times \left[ n \frac{\gamma+1}{n+1} \left( \frac{z_0^{n+1}}{2n+\gamma n+1} \right)^{1/n} \right]^{1/(\gamma+1)}. \end{aligned}$$

This solution describes the propagation of the magnetic field into the sample, the magnetic induction being localized in the domain between the surface  $x = 0$  and the flux front  $x_p$ . This solution is positive in the plane  $x_p > x$  and is zero outside of it. Note that only the  $x > 0$  and  $t > 0$  quarter of the plane is presented, because of its physical relevance. The penetrating flux front position  $x = x_p(t)$  as a function of time can be described by the relation

$$x_p = x_0 t^{-1/(2n+\gamma n+1)}.$$

Using the last relation the velocity of the magnetic flux induction can be obtained as the following

$$v_p \approx v_0 t^{-n(2+\gamma)/(2n+\gamma n+1)}. \quad (21)$$

Let us now consider the most interesting case  $n = 1$ . In this particular case the spatial and temporal evolution of the magnetic flux induction is totally determined by the parameters  $\gamma$ ,  $\alpha$  and  $\beta$ . As the following analysis we may derive an evolution equation for the magnetic induction profile for the case  $n = 1$  and apply the scalings of the previous section to formulate a similarity solution for the  $b(x, t)$ . The time and space evolution of the self-similar solution of magnetic field penetration into a superconductor for  $n = 1$  is shown schematically in Figure 1(a) and (b).

## 5 Conclusion

Thus, in this paper, we have solved the nonlinear diffusion equation analytically to provide expressions for the time-space evolution of the magnetic induction for different values of exponents  $n$ ,  $q$  and  $\alpha$ . For a given parameter set  $n$ ,  $q$  and  $\alpha$  the form of the scaling function  $f(z)$  has been obtained by solving the nonlinear diffusion equation analytically by a self-similar technique. The spatial and temporal profiles of magnetic flux penetration in the sample depends on the set of three independent parameters,  $n$ ,  $q$  and  $\alpha$ . It is of interest to consider the nonlinear diffusion equation for the magnetic induction for different values of the exponents  $n$ ,  $q$  and  $\alpha$ . The obtained solution can be experimentally detected as an elementary one-dimensional structure, for example dendritic branches or fingering patterns in a varying magnetic field [7].

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